Statistical Mechanical Interpretation of Black Hole Entropy

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It is shown that the Bekenstein-Hawking formula for the entropy of a black hole can be given a statistical mechanical interpretation in terms of Planck mass particles. It is furthermore shown that the previously proposed Planck aether model (assuming that space is densely filled with an equal number of positive and negative Planck masses) gives an expression for the black hole entropy, different from the Bekenstein-Hawking formula, with the entropy proportional to the 3/4 power of the black hole surface rather than proportional to its surface.

The Planck aether model also gives an expression for the entropy of the gravitational field, which for a black hole is the entropy of negative Planck masses. To be consistent with Nernst's theorem, it is conjectured that this gravitational field entropy is negative. For a universe in which the sum of the positive matter energy and the negative gravitational field energy is zero, the sum of the matter and gravitational field entropy would therefore vanish as well. Because the positive and negative Planck masses are separated from each other, a cancellation of their entropy appears to be only possible in the event of a gravitational collapse of the universe as a whole.

1. Introduction

According to Dyson [1], the Bekenstein-Hawking formula for the entropy of a black hole (in which the entropy of a black hole is proportional to its surface) is as great a mystery as was Planck's equation, $E = h\nu$, postulating a proportionality between energy and frequency. The Bekenstein-Hawking entropy formula [2, 3] is (up to a numerical constant of the order unity) given by

$$S = A k c^3/G\hbar\,, \tag{1.1}$$

where A is the surface of the black hole (k Boltzmann constant, c velocity of light, G Newton's constant, and $\hbar$ Planck's constant). As long as one is only interested in the order of magnitude for S, one can set

$$A = R_0^2\,, \tag{1.2}$$

where

$$R_0 = \frac{GM}{c^2} \tag{1.3}$$

is the gravitational radius of a mass M.

Introducing the Planck length

$$r_p = \sqrt{\frac{G\hbar}{c^3}}\,, \tag{1.4}$$

one can write for (1.1)

$$S(R_0/r_p)^2 k\,. \tag{1.5}$$

For a solar mass black hole one has $R_0 \sim 10^5$ cm. With $r_p \sim 10^{-33}$ cm, the Bekenstein-Hawking entropy (in units in which $k=1$) gives $S \sim 10^{76}$.

By comparison, the statistical mechanical expression for an assembly of N particles is (up to a logarithmic factor) given by

$$S = N k\,. \tag{1.6}$$

Containing about $\sim 10^{57}$ baryons, the statistical mechanical entropy of the sun would be $S \sim 10^{57}$, that is 19 orders of magnitude smaller than the entropy given by (1.5). This large discrepancy makes it difficult to understand the Bekenstein-Hawking black hole entropy within the framework of statistical mechanics. It is the purpose of this communication to show that the previously proposed Planck aether model of the vacuum can provide such an explanation.

2. The Planck Aether Hypothesis

Quantum mechanics, in conjunction with the postulates of special relativity, inevitably leads to divergencies for which no convincing cure has been found to date. If special relativity is interpreted as a space-time symmetry, causality demands that elementary particles must have the structure of mathematical points. Because of Heisenberg's uncertainty principle, the point-like structure not only leads to divergent selfenergies but also to a divergent zero point vacuum

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energy. To be relativistically invariant, this vacuum energy must have a divergent ω^3 frequency spectrum. It is, however, clear that this spectrum must have at least a cut-off at the Planck energy. A cut-off at any energy, including the Planck energy, would destroy the relativistic invariance of the zero point energy spectrum, and it would introduce a preferred reference frame in which the zero point energy is at rest. Another more serious problem with a cut-off at the Planck energy is that it would lead to a huge vacuum mass density, of the order $\sim 10^{95}$ g/cm^3, large enough to put the mass of the entire universe into a cube with a side-length of one fermi.

In an attempt to overcome the problem of the large mass density, it was suggested by Sakharov [4] that space is densely filled with hypothetical particles, which he called maximons, with a mass equal a Planck mass, in addition to an equal number of ghost particles to compensate the large mass density. Sakharov's idea comes close to what I have called the Planck aether hypothesis. It assumes that space is densely filled with an equal number of positive and negative Planck masses. The vacuum is thereby given the appearance of "mass neutrality" in analogy to electric charge neutrality of condensed matter. There are, however, two important differences between Sakharov's idea and the Planck aether hypothesis.

1) Whereas in Sakharov's hypothesis the maximons possess gravitational charge and are the source of a gravitational field, the Planck masses in the Planck aether hypothesis have no gravitational or any other charge. The only interact with each other locally.

2) Whereas in Sakharov's hypothesis the maximons obey a relativistic law of motion, the Planck masses in the Planck aether hypothesis obey an exactly nonrelativistic law of motion.

Because in a nonrelativistic field theory the particle number operator commutes with the Hamilton operator, the Planck masses are conserved as nondestructive elements or Leibnizian monads. It is for this reason that the entirety of the Planck masses form a two-component positive-negative mass superfluid, taking somehow the place of the pre-relativity aether models of classical physics*. The assumption of local contact-type interactions follows the spirit of Heisenberg's nonlinear spinor theory, where the spinors have no charges to act as sources of fields. The long-range interactions are there the result of the local contact-type interactions. The same is true for the Planck aether model.

To describe the Planck aether mathematically, one has to search for a nonlinear, nonrelativistic Heisenberg-type operator field equation. With simplicity as a guide, I have chosen the two-component operator field equation [5]

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_\text{p}} \nabla^2 \psi_\pm \pm 2\hbar c r_\text{p}^2 (\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp)\, \psi_\pm , \tag{2.1}$$

where $\psi_\pm$ have to obey the canonical commutation relations

$$\begin{aligned} [\psi_\pm(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] &= \delta(\boldsymbol{r}-\boldsymbol{r}') , \\ [\psi_\pm(\boldsymbol{r})\, \psi_\pm(\boldsymbol{r}')] &= [\psi_\pm^\dagger(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] = 0 . \end{aligned} \tag{2.2}$$

In (2.1), $m_\text{p} = \sqrt{\hbar c/G} \simeq 2.2 \times 10^{-5}$ g and $r_\text{p} = \sqrt{\hbar G/c^3} \simeq 1.6 \times 10^{-33}$ cm are the Planck mass and Planck length. Making the Hartree approximation, the field operators are replaced by their expectation values and the product of three field operators by the products of their expectation values. This way one obtains the classical field equation for (2.1), which is the same as its quantized version. In the more accurate Hartree-Fock approximation taking into account the completely symmetric wave function of the groundstate for each Planck mass component, the exchange interactions between equal Planck masses make an equal contribution, leading to the nonlinear Schrödinger equation

$$i\hbar \frac{\partial \varphi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_\text{p}} \nabla^2 \varphi_\pm \pm 2\hbar c r_\text{p}^2 (2\varphi_\pm^* \varphi_\pm - \varphi_\mp^* \varphi_\mp)\, \varphi_\pm , \tag{2.3}$$

where $\varphi = \langle\psi\rangle$, $\varphi^* = \langle\psi^\dagger\rangle$, $\langle\psi_\pm^\dagger \psi_\pm \psi_\pm\rangle \simeq 2\varphi_\pm^* \varphi_\pm^2$, $\langle\psi_\mp^\dagger \psi_\mp \psi_\pm\rangle \simeq \varphi_\mp^* \varphi_\mp \varphi_\pm$, with the factor 2 in the expectation value for the product of operators belonging to the same particle species coming from the exchange integral of the delta-function type contact interaction between equal particles.

By putting

$$\begin{aligned} n_\pm &= \varphi_\pm^* \varphi_\pm , \\ n_\pm v_\pm &= \mp \frac{i\hbar}{2m_\text{p}} [\varphi_\pm^* \nabla \varphi_\pm - \varphi_\pm \nabla \varphi_\pm^*] , \end{aligned} \tag{2.4}$$

* It must be emphasized though that the aether in pre-relativity physics was a hypothetical substance assumed to exist in addition to matter, whereas the Planck aether has to be understood as a Heisenberg-type fundamental field from which all elementary particles and their interactions are derived.

(2.3) can be brought into its hydrodynamic form

$$\frac{\partial \boldsymbol{v}_\pm}{\partial t} + (\boldsymbol{v}_\pm \cdot \nabla)\, \boldsymbol{v}_\pm = -2c^2 r_p^3\, \nabla(2n_\pm - n_\mp) + \frac{1}{m_p} \nabla Q_\pm\,,$$

$$\frac{\partial n_\pm}{\partial t} + \nabla(n_\pm \boldsymbol{v}_\pm) = 0\,, \tag{2.5}$$

where

$$Q_\pm = \frac{\hbar^2}{2m_p} \frac{\nabla^2 \sqrt{n_\pm}}{\sqrt{n_\pm}} \tag{2.6}$$

is the quantum potential, and where we made use of $m_p r_p c = \hbar$. The solutions of (2.5) consist of quantized vortices and compression waves**.

In the Planck aether model, gravitational mass has its origin in Planck masses bound in quantized vortex filaments. The zero point kinetic energy density of Planck masses bound in the quantized vortex filaments of radius r_p is of the order $\hbar c/r_p^4 = G m_p^2/r_p^4$, which is of the same order of magnitude as the gravitational field energy $g^2/8\pi$, the gravitational field a Planck mass would have at the distance $r = r_p$ with $g = \sqrt{G}\, m_p/r^2$. Because of the zero point oscillations of the Planck masses bound in the vortex filaments, they become the source of virtual phonons, leading to an attractive inverse square force law with respect to other likewise bound Planck masses. The origin of the inertial mass, and hence the equivalence of inertial and gravitational mass, results from the interaction with all vortex filaments in the universe containing bound Planck masses, very much as in Mach's principle.

If the density of the vortex filaments in the superfluid Planck aether is large, in what has been called a vortex sponge, two transverse types of wave modes are possible: A mode which can describe Maxwell's vector field equation, and a mode which can describe Einstein's tensor field equation. The choice of the coupling constant in the nonlinear term of (2.1) makes these waves propagate with the velocity of light. Because the Planck aether consists of two superfluids, one which can assume positive and the other one negative kinetic energies, a vortex sponge of positive and negative vortices can form without the expenditure of energy.

A vortex sponge has a cut-off for these waves at a wavelength smaller than the distance of separation between the vortex filaments. In hydrodynamics, an equilibrium distance is determined by the minimum drag Reynold's number $\mathrm{Re} \lesssim 10^7$. For this equilibrium, $\sqrt{\mathrm{Re}} \sim 3 \times 10^3$ is the ratio of the distance of separation between the vortex filaments and their core radius. With the core radius equal the Planck length, the distance of separation is $\sim 10^{-29}$ cm. It turns out to have a scale which by order of magnitude corresponds to the energy (GUT energy) at which the strong, electromagnetic and weak interaction are unified.

If the vortex sponge is dense enough, a lattice of vortex rings is likely to form by the process of colliding and reconnecting vortex filaments. The vortex rings have a sharp resonance at about $\sim 10^{12}$ GeV. Two such resonances, one having a positive and the other one having a negative energy, can form an excitonic soliton whereby the resulting positive mass slightly exceeds the negative mass due to the positive gravitational interaction energy of two masses having opposite sign. It turns out that such excitonic solitons not only have the property of Dirac spinors, but that their mass expressed in terms of the Planck mass is

$$m/m_p \sim \mathrm{Re}^{-3} \sim 10^{-21} \tag{2.7}$$

in rough agreement with the empirical, mass ratio of a typical spinor mass to the Planck mass. These Dirac spinors are held together in a static equilibrium through interactions communicated by waves, which in the rest frame of the Planck aether propagate with the velocity of light. Therefore, all material objects composed of Dirac spinors and in absolute motion against the Planck aether suffer a true contraction equal to the Lorentz contraction. Because all clocks behave under these circumstances as light clocks, the contraction effect alone is sufficient to reproduce Einstein's time dilation effect [6]. Special relativity can therefore be understood as a purely dynamic effect in absolute space and absolute time, with the Minkowskian space-time metric seen as an illusion caused by an equal deformation of all bodies in absolute motion.

Relativistic invariance as a dynamic symmetry breaks down if the quasiparticles of the Planck aether (of which ordinary matter is composed) approach the Planck energy. Above this energy, physical reality

** The previously used Hartree-approximation is insufficient for a self-consistent description of the superfluid vortex dynamics. To obtain time independent vortex solutions in the Hartree approximation, one must in addition assume that in lowest order the mutual interaction of the two superfluid components can be neglected. No such additional assumption is needed in the Hartree-Fock approximation.

is better described by Newtonian nonrelativistic mechanics for the Planck masses. For an elementary particle of mass m to reach the Planck energy and to disintegrate, it must have an absolute velocity given by

$$m/m_p = \sqrt{1 - v^2/c^2}\,, \tag{2.8}$$

hence

$$v/c = \sqrt{1 - (m/m_p)^2}\,. \tag{2.9}$$

Because Dirac spinors are excitonic solitons made up from the resonance energy of two vortex rings with opposite mass, their breakup results in the formation of free vortex rings which ultimately can become rotons. According to Feynman, rotons can be visualized as vortex rings with a ring radius equal to the vortex core radius, with the vortex radius in the Planck aether equal the Planck length. It is for this reason that Planck masses bound in rotons have the same zero point energy as those confined in vortex filaments. As a result of their zero point energy fluctuations, Planck masses bound in vortex rings or rotons assume gravitational mass. The vortex rings and rotons form a gravitationally interacting gas, which is in an excited state above the gravitationally noninteracting superfluid groundstate. It is with these excited Planck mass objects that we try to understand black hole entropy in terms of statistical mechanics.

3. The Planck Aether Hypothesis and the Physics of Gravitational Collapse

Schwarzschild's solution of Einstein's gravitational field equation expressed by the line element

$$ds^2 = \frac{dr^2}{1 - \dfrac{2GM}{c^2 r}} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) - c^2\left(1 - \frac{2GM}{c^2 r}\right) dt^2 \tag{3.1}$$

can be given a simple interpretation in terms of special relativity and Newtonian gravity. According to Newtonian mechanics, a body falling in the gravitational field of a mass M assumes a velocity v given by

$$v^2 = \frac{2GM}{r}\,. \tag{3.2}$$

In combination with special relativity, it leads to the length contraction

$$dr = dr'\sqrt{1 - \frac{v^2}{c^2}} = dr'\sqrt{1 - \frac{2GM}{c^2 r}} \tag{3.3}$$

and the time dilation

$$dt = dt'/\sqrt{1 - v^2/c^2} = dt'/\sqrt{1 - 2GM/c^2 r}\,. \tag{3.4}$$

Inserting (3.3) and (3.4) into the line element of special relativity $ds^2 = dr'^2 - c^2\, dt'^2$, one obtains Schwarzschild's line element (3.1). Unlike dr and dt, which are measured at $r \approx \infty$ by an outside observer far away from the mass M, dr' and dt' are measured in the inertial system carried along with the infalling body. According to (3.2), the body reaches the velocity $v = c$ at the Schwarzschild radius $R_S = 2GM/c^2 = 2R_0$. The time needed to reach $r = R_S$ is, for the outside observer, infinite, even though it is finite in the inertial system carried along with the infalling body. Since the same holds true for a body as a whole, collapsing under its own gravitational field, the gravitational collapse time for an outside observer is infinite. In the Planck aether model, this time is finite because in approaching $r = R_S$ the kinetic energy of all the elementary particles, which the body is composed of, rises in proportion to $1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - 2GM/c^2 r}$, becoming infinite at $r = R_S$. If this energy reaches the Planck energy at a radius somewhat larger than R_S, the elementary particles disintegrate into Planck mass objects as described above*.

In approaching the Schwarzschild radius R_S, an infalling particle assumes relativistic velocities, which permits us to set in (3.1) $ds = 0$. The velocity seen by an outside observer therefore is approximately given by

$$\frac{dr}{dt} = -c\left(1 - \frac{R_S}{r}\right) \tag{3.5}$$

with the velocity of the infalling body measured in its own inertial reference system about equal to c. From (3.5) one has

$$-ct = \int \frac{dr}{1 - R_S/r} \simeq R_S \int \frac{dr}{r - R_S} \tag{3.6}$$

* Strictly speaking, this is only true if the gravitational collapse takes place in a system at rest with the Planck aether. According to the dynamic interpretation of special relativity, the Lorentz contraction is only in this reference system a real physical effect. Since most black holes are (from a relativistic perspective) nearly at rest relative to the system of galaxies, with the system of galaxies likely to be at rest with the hypothetical Planck aether, the assumption made that the energy of the infalling particles relative to the Planck aether is proportional to $1/\sqrt{1 - 2GM/c^2 r}$ is a very good one.

and hence

$$r - R_S = \text{const} \cdot e^{-ct/R_S}. \quad (3.7)$$

If at $t=0$, $r=(a+1)\,R_S$, $a \gg 1$, it follows from (3.7) that

$$\frac{r-R_S}{R_S} = a\,e^{-ct/R_S}. \quad (3.8)$$

Because of (2.8) and (3.2) one must have

$$m/m_p = \sqrt{1-v^2/c^2} = \sqrt{1-R_S/r}, \quad (3.9)$$

hence

$$\frac{r-R_S}{R_S} \simeq \left(\frac{m}{m_p}\right)^2. \quad (3.10)$$

Inserting this value into (3.8) and solving for $t = t_0$, the time needed to reach the distance r at which the kinetic energy of an infalling particle becomes equal the Planck energy, one finds

$$t_0 = \frac{R_S}{c} \ln\left[a\left(\frac{m_p}{m}\right)^2\right]. \quad (3.11)$$

After having disintegrated into Planck mass objects, that is free rotons or vortex rings, and passed the Schwarzschild radius $r=R_S$ at which $v=c$, the time needed to reach $r=0$ is of the order R_S/c. The logarithmic factor therefore represents the increase of the gravitational collapse time over the nonrelativistic value R_S/c. Only in the limit $m_p \to \infty$ does this time become infinitely long as in general relativity. For an electron, one has $m_p/m \sim 10^{22}$ and hence

$$\ln\left(\frac{m_p}{m}\right)^2 \simeq 100, \quad (3.12)$$

making the collapse time in the Planck aether model about 100 times longer than in the nonrelativistic case. In the example of a solar mass black hole one has $R_S \sim 1$ km, and hence for the nonrelativistic collapse time $R_S/c \sim 3 \times 10^{-6}$ sec. The gravitational collapse time for a solar mass black hole, with the collapse starting from $r = 100\,R_S (a \sim 10^2)$, would therefore be $\sim 100\,R_S/c \sim 3 \times 10^{-4}$ sec. It therefore would follow that the relativistic time retardation for the gravitational collapse is, from an astronomical point of view, completely insignificant.

During the gravitational collapse, the location of the event horizon, defined as the position at which $v=c$, develops from "inside out", both in Newtonian and Einsteinian gravity. Assuming a spherical body of radius R and constant density ϱ, the Newtonian potential inside and outside the body is

$$\begin{aligned}\phi_{in} &= -\frac{GM}{2R}\left[3-\left(\frac{r}{R}\right)^2\right], \quad r<R,\\ \phi_{out} &= -\frac{GM}{r}, \quad r>R.\end{aligned} \quad (3.13)$$

An infalling test particle would reach at $r=0$ the maximum velocity v given by

$$\frac{v^2}{2} = -\phi_{in}(0) = \frac{3}{2}\frac{GM}{R}. \quad (3.14)$$

Therefore, it could reach the velocity of light if the sphere has contracted to the radius

$$R_1 = 3\,GM/c^2 = (3/2)\,R_S = 3\,R_0. \quad (3.15)$$

Accordingly, the disintegration of the matter inside the sphere begins at its center after the sphere has collapsed to (3/2) the Schwarzschild radius.

A qualitatively similar result is obtained from general relativity by taking the Schwarzschild interior solution for an incompressible fluid [7]. The event horizon begins there to develop at $r=0$, in the moment the sphere has contracted to the radius

$$R_1 = (9/8)\,R_S = (9/4)\,R_0. \quad (3.16)$$

4. Interpretation of the Bekenstein-Hawking Entropy

If the negative gravitational energy of a spherical body of mass M and radius R, which by order of magnitude is $-GM^2/R$, is set equal its rest mass energy Mc^2, the body has contracted to the gravitational radius $R_0 = GM/c^2$. After a body has passed through the event horizon, its internal energy is confined and cannot be lost by radiation or otherwise. In passing through the event horizon, the matter of the collapsing body has disintegrated into Planck mass objects, the number of which is obtained from

$$N_0\,m_p\,c^2 = \frac{GM^2}{R_0} = M\,c^2 \quad (4.1)$$

and therefore

$$N_0 = M/m_p. \quad (4.2)$$

For a solar mass $M \sim 10^{33}$ g (with $m_p \sim 10^{-5}$ g), one has $N_0 \sim 10^{38}$. Using units in which $k=1$, one would have at the beginning of the collapse, $S \sim N_0 \sim 10^{38}$.

If the collapse proceeds to a radius $r < R_0$, the number of Planck masses N increases according to

$$N m_p c^2 = \frac{GM^2}{r} \tag{4.3}$$

or as

$$N/N_0 = (M/m_p)(R_0/r). \tag{4.4}$$

This increase is possible because the vacuum has an unlimited supply of Planck masses in the superfluid groundstate. With the negative energy of the gravitational field interpreted in the Planck aether model by an excess of negative over positive Planck mass objects, the gravitational collapse below $r = R_0$ implies that a pair of positive and negative Planck mass objects (that is rotons or vortex rings) are generated out of the superfluid groundstate, with the positive Planck masses accumulating in the collapsing body, and with the negative Planck masses, from which the positive Planck masses have been separated, surrounding the body.

One can then ask to what radius the body has to collapse to make the Bekenstein-Hawking entropy (1.5) equal to the statistical mechanical value (1.6). Equating (1.5) with (1.6), expressing N with (4.3) and solving for r, one finds for the collapse radius

$$r = r_p. \tag{4.5}$$

Inserting $r = r_p$ into (4.3), one finds $N \sim (R_0/r_p)^2$. That many positive Planck mass objects would have to be confined within a Planck radius to account for the Bekenstein-Hawking entropy. For a solar mass black hole, this number and entropy would be $N \sim S \sim 10^{76}$.

5. Black Hole Entropy in the Planck Aether Model

The Planck mass objects, consisting of rotons or vortex rings, do not interact gravitationally for a distance of separation smaller than a Planck length. It is for this reason that a spherical assembly of N Planck masses bound in vortices resp. rotons cannot collapse through the action of gravitational forces below a radius smaller than

$$r_0 = N^{1/3} r_p. \tag{5.1}$$

The core of a solar mass black hole, for example, would have a radius $r_0 \sim 10^{19} r_p \sim 10^{-14}$ cm. Inserting the value (5.1) into (4.3), one has

$$N = (R_0/r_p)^{3/2}. \tag{5.2}$$

In statistical mechanics, the entropy of a gas composed of N Planck mass particles m_p with a temperature T is given by the Sakur-Tetrode formula [8]

$$S = N k \ln[(V/N)(2\pi m_p kT)^{3/2} e^{5/2}/\hbar^3], \tag{5.3}$$

where $V/N = r_p^3$, with the smallest phase space volume $(\Delta p\, \Delta q)^3 = (m_p c \cdot r_p)^3 = \hbar^3$. The temperature T in the core of the black hole is the Davies-Unruh temperature [3, 9, 10]

$$T = \hbar a / 2\pi k c, \tag{5.4}$$

where $a = GM/r_0^2 = c^2 R_0/r_0^2$ is the gravitational acceleration at $r = r_0$. With the help of (5.1) and (5.2), one obtains from (5.3)

$$S = (5/2)(R_0/r_p)^{3/2} k = (5/2) N k. \tag{5.5}$$

The entropy of a black hole in the Planck aether model therefore goes in proportion of the (3/4) power of the black hole area, instead in proportion of the area as in the Bekenstein-Hawking entropy.

It was pointed out by Bekenstein [2] that the black hole entropy cannot be proportional to the radius of the black hole. The mass of a black hole is proportional to its radius, and the merger of two black holes would for this reason lead to an entropy equal the sum of the entropies for the two holes. With the merger of two black holes, obviously an irreversible process, this would contradict the second law of thermodynamics. In the Bekenstein-Hawking formula, the entropy is porportional to the area of the black hole, with the entropy of two merged black holes larger than their sum. But the same is still true if the entropy is proportional to the (3/4) power of the area as it is suggested by the Planck aether model. In this model, the entropy of two black holes with area A_1 and A_2 prior to their merger would be proportional to

$$S_0 = A_1^{3/4} + A_2^{3/4} \tag{5.6}$$

and after the merger proportional to

$$S_1 = A^{3/4},$$

where

$$A^{3/4} = (A_1^{1/2} + A_2^{1/2})^{3/2} > A_1^{3/4} + A_2^{3/4} \tag{5.7}$$

as required by the second law.

With the black hole entropy proportional to the (3/4) power of its area, the entropy of a solar mass black hole is reduced from the Bekenstein-Hawking value $S \sim 10^{76}$ to $S \sim 10^{57}$. This is about equal the statistical mechanical value if N is set equal the number of baryons in the sun.

6. The Entropy of the Universe

For a flat universe (suggested by inflationary models but also by observational evidence), the sum of the positive energy of matter and negative gravitational potential energy must vanish. In keeping with our order of magnitude estimates, we may express this statement by equating the rest mass energy of the universe Mc^2 (where M is the mass of the universe) with the (negative) gravitational potential energy $2GM^2/R$, where R is the radius of the universe, resulting in

$$R = 2GM/c^2 . \tag{6.1}$$

Because this turns out to be the same relation as the one for the Schwarzschild radius of a mass M, we may simply apply the expressions derived for the entropy of a black hole to the universe.

The universe contains $\sim 10^{80}$ baryons and it has a mass $M \sim 10^{56}$ gram. It, therefore, has a gravitational radius equal to $R_0 \sim 10^{28}$ cm. With these values, the Bekenstein-Hawking entropy of the universe is $S \sim 10^{121}$. In the Planck aether model, the entropy would be reduced to $S \sim 10^{91}$. The entropy of the baryons in the universe is about equal their number $\sim 10^{80}$. A much larger contribution to the entropy comes from the photons of the cosmic microwave background radiation. With $\sim 10^8$ photons per baryon, the entropy of this radiation is $\sim 10^{88}$. On a logarithmic scale, this value is rather close to (5.5).

7. The Entropy of the Gravitational Field

If the total energy of the universe is zero, one must ask why is its entropy so large? It seems that in the course of a gravitational collapse, a mixing of the positive kinetic energy with the negative gravitational energy would lead to a state of zero temperature. According to Nernst's theorem, the entropy of the collapsed state would, therefore, have to be zero. Since the same would have to be true for the state from which the cosmological expansion began, the entropy at the beginning of the expansion would have to vanish as well. The very large entropy of the universe according to the Bekenstein-Hawking formula of the order $\sim 10^{121}$, but even according to the Planck aether hypothesis still $\sim 10^{91}$, requires an extremely small initial phase space volume from which the expansion would have to start. It was pointed out by Penrose [11] that the extremely small initial phase space volume presents a serious problem (excluding an act of "divine" intervention) for the inflationary and similar models. For the Bekenstein-Hawking entropy, the initial phase space volume would amount to a fraction of the total phase space volume given by

$$f = e^{-S} \sim \exp\left[-10^{121}\right] \tag{7.1}$$

and in the Planck aether model by

$$f = e^{-S} \sim \exp\left[-10^{91}\right] . \tag{7.2}$$

In the Planck aether model, the negative gravitational field energy surrounding a mass is due to an excess of a negative over a positive mass. According to (4.3), the negative mass surrounding a highly collapsed spherical body must be of the same order of magnitude as the positive mass accumulated inside the collapsing body. An assembly of interacting positive and negative masses can, in general, not lead to a thermodynamic equilibrium. But because in the state of maximum collapse down to the radius $r_0 = N^{1/3} r_p$, all the positive masses are separated from the negative ones, it is sufficient to require that each mass species reaches thermodynamic equilibrium. It was shown by Vysin [12] that an assembly of negative masses can acquire thermodynamic equilibrium provided the temperature is negative. The kinetic energy of a negative Planck mass is negative, and an assembly of negative Planck masses has, for this reason, a negative temperature. It therefore can reach thermodynamic equilibrium.

We now make the following hypothesis:

If an assembly of positive and negative masses, with the total energy equal to zero, is brought together, the temperature and hence entropy of the mixture will go to zero.

This hypothesis is the only one consistent with Nernst's theorem. It, of course, implies that an assembly of positive and negative masses can perfectly mix because otherwise no equilibrium can be reached. To satisfy the hypothesis, we assume that the negative masses have negative entropy. Only the assumption that an assembly of negative masses has a negative entropy permits an analytic continuation of the entropy from positive to negative temperatures as shown in Figure 1. If the entropy for negative temperatures would be positive (as it was assumed by by Vysin [12] and shown by the dashed line for negative temperatures), the function dS/dT would be discontinuous at $T = 0$.

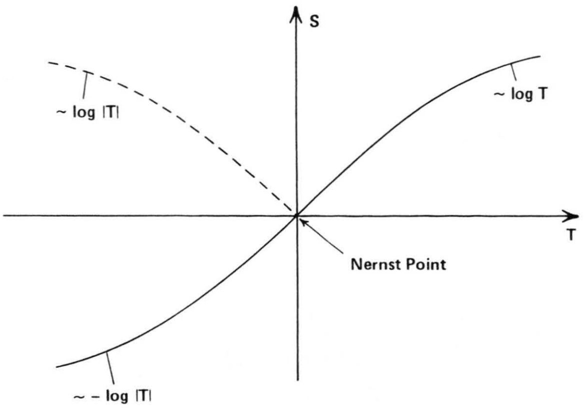

Fig. 1. Entropy as a function of the temperature, analytically continued to negative temperatures (dashed line is the non-analytic continuation proposed by Vysin [12]).

For the entropy of a mixture of positive and negative masses to become zero requires an exact correlation in the disorder of the positive mass gas with the disorder of the negative mass gas. This is certainly true if the negative mass is equal to the negative mass of the gravitational field of the positive mass, because the Newtonian gravitational field of each particle, all the way down to the smallest dimension, is precisely correlated to the position of the particle. The entropy of the positive mass of matter and the entropy of the negative mass of its gravitational field (correlated to the entropy of the positive mass which is the source of this field) might therefore be called complementary. The expansion from a very small phase space volume would then be possible, because if the positive and negative masses are densely packed within the same volume, not only their energy, but also their entropy would cancel.

The assignment of a negative temperature to the negative masses surrounding a gravitationally collapsed body also removes another paradox. According to the principle of equivalence, also fully valid in the Planck aether model, a positive mass attracts all masses and likewise, a negative mass repels all masses. It therefore would seem that the positive mass of a gravitationally collapsed body would attract the negative mass of the gravitational field surrounding the positive mass, with the result that the negative mass would annihilate the positive mass of the collapsed body. But if the gravitational field surrounding the body has a temperature and entropy, the pressure generated by it would counteract the attractive force exerted by the collapsed body. To obtain a value of the negative temperature of the gravitational field surrounding a Schwarzschild black hole, we may take the Davies-Unruh formula (5.4). At the surface of the Schwarzschild sphere, one has $a \sim GM/R_0^2 = c^2/R_0$ and hence

$$k\,T \sim -\hbar c/R_0\,. \tag{7.3}$$

This is the minimum negative temperature permitted by the uncertainty principle for a negative zero rest mass particle confined within a Schwarzschild sphere. It therefore follows that the collapse of the gravitational field onto the mass which generates the field is forbidden for the same reason, which forbids the collapse of the electron cloud in an atom onto its nucleus.

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